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Warmup

Find the foci

$$1. \frac{(x+6)^2}{36} - \frac{(y+3)^2}{9} = 1$$

$$(-6 \pm 3\sqrt{5}, -3)$$

$$2. \frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$$

$$(2, 3 \pm \sqrt{41})$$

$$3. \frac{(y-4)^2}{16} - \frac{(x+2)^2}{9} = 1$$

$$(-2, 4 \pm 5)$$

$$4. \frac{(x+1)^2}{4} - \frac{(y-4)^2}{9} = 1$$

$$(-1 \pm \sqrt{13}, 4)$$



Compound Interest

Compounded Interest

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$$\begin{array}{c} \text{Initial} \\ \text{Investment} \\ \downarrow \\ \text{Future Value} \rightarrow V_n = P(1 + r)^n \\ \uparrow \\ \text{Interest Rate} \end{array} \quad \begin{array}{c} \text{Number} \\ \text{of time} \\ \text{periods} \end{array}$$

An initial investment of \$25,000 is made with a yearly interest rate of 5%.
What is the value after 6 years?

$$V_n = P(1 + r)^n$$

$$V_6 = 25000(1 + 0.05)^6$$

$$V_6 = 25000(1.05)^6$$

$$V_6 = 25000(1.34)$$

$$V_6 = \$33,502.39$$

What if it were compounded monthly?

$$V_6 = 25000 \left(1 + \frac{0.05}{12} \right)^{6 \cdot 12}$$

\$33,725.44

Compounded Interest

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Deposit \$1,000 with a yearly interest rate of 5% compounded monthly.

What is the value after a year?

$$A(t) = A_0(1 + r)^t \qquad A(12) = 1000 \left(1 + \frac{0.05}{12} \right)^{12} \approx \$1,051.16$$

Practice

Invest \$30,000 at 4.5% interest per year. What is the value after one year?

- | | | |
|--|--|-----------------------|
| 1) Interest is compounded monthly | $30000 \left(1 + \frac{0.045}{12} \right)^{12}$ | $\approx \$31,378.19$ |
| 2) Interest is compounded weekly
(1 yr = 52 wks) | $30000 \left(1 + \frac{0.045}{52} \right)^{52}$ | $\approx \$31,380.23$ |
| 3) Interest is compounded daily
(1 yr = 365 days) | $30000 \left(1 + \frac{0.045}{365} \right)^{365}$ | $\approx \$31,380.75$ |

Compounded Interest

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Euler's Constant - continuous interest

Invest \$30,000 at 4.5% interest per year. What is the value after one year?

If compounded continuously?

$$A(t) = A_0(e)^{rt}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e = 2.71828...$$

$$30000e^{0.045 \cdot 1} \approx \$31,380.84$$

Practice

Invest \$20,000 at 6.5% interest per year. What is the value after one year?

1) Interest is compounded monthly

$$20000 \left(1 + \frac{0.065}{12}\right)^{12} \approx \$21,339.44$$

2) Interest is compounded weekly

$$20000 \left(1 + \frac{0.065}{52}\right)^{52} \approx \$21,342.31$$

3) Interest is compounded continuously

$$20000e^{0.065 \cdot 1} \approx \$21,343.18$$

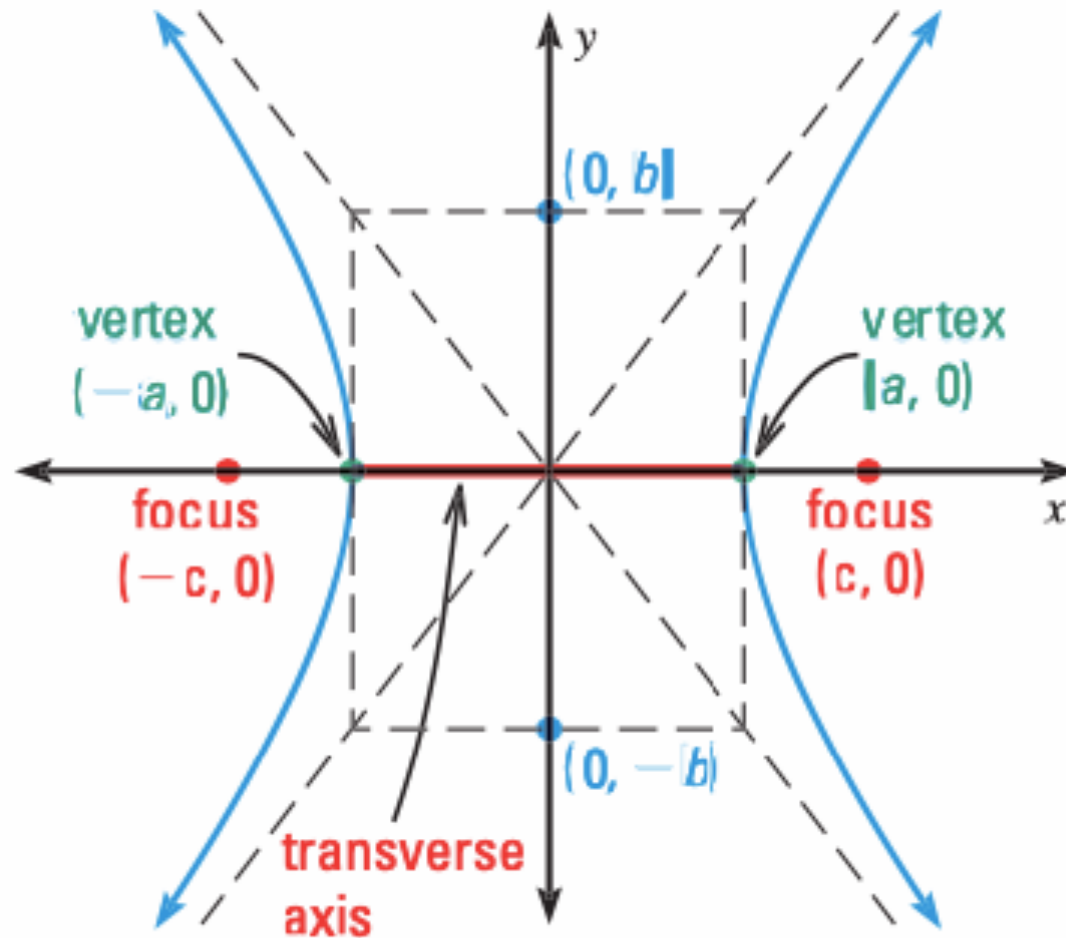


More on Hyperbolas

Conics - Hyperbola

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Definition



Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

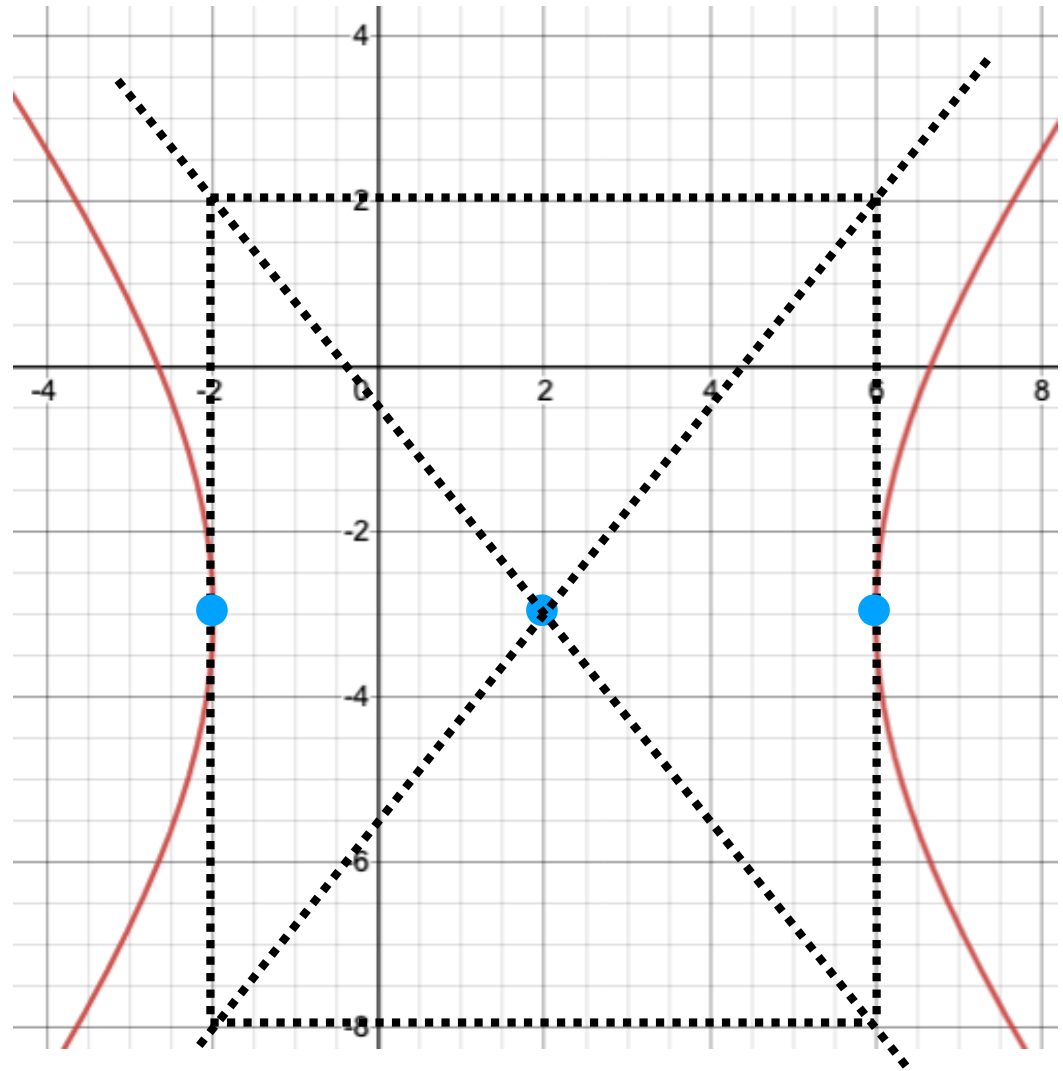
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Graph and find the foci.

$$\frac{(x - 2)^2}{16} - \frac{(y + 3)^2}{25} = 1$$

$$(2 \pm \sqrt{41}, -3)$$



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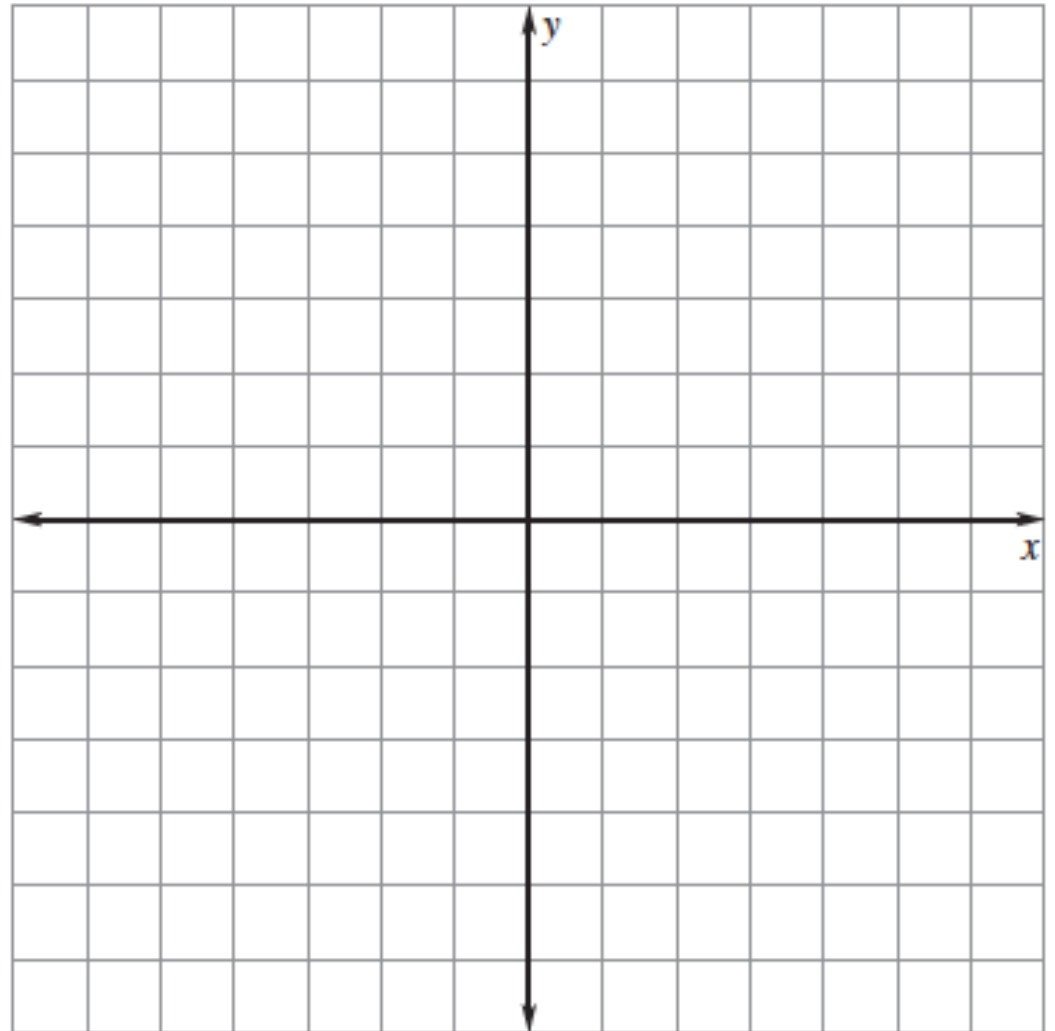
Find significant features and graph the following

$$9x^2 + 18x - 4y^2 + 16y - 43 = 0$$

Center: $(-1, 2)$

Foci: $(-1 \pm \sqrt{13}, 2)$

Asymptotes: $y = \frac{3}{2}x + \frac{7}{2}$
 $y = -\frac{3}{2}x + \frac{1}{2}$



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Find the equation with the given.

Asymptotes: $y = \frac{5}{2}x - 7$
 $y = -\frac{5}{2}x + 3$

Opens vertically

$$a = 10$$

$$\frac{(y + 2)^2}{100} - \frac{(x - 2)^2}{16} = 1$$



More on Logarithms

6.5 - Properties of Logarithms

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Logarithm Property

$$\frac{1}{\log_x y} = \frac{1}{\frac{\log y}{\log x}} = \frac{\log x}{\log y} = \log_y x$$

$$\frac{1}{\log_5 10} = \log_{10} 5$$

$$\log_2 7 = \frac{1}{\log_7 2}$$

6.5 - Properties of Logarithms

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Changing bases

$$y = 9^{\log_2 81}$$

$$\log_9 y = \log_2 81$$

$$\frac{\log_2 y}{\log_2 9} = \log_2 81$$

$$\log_2 y = \log_2 9 \cdot \log_2 81$$

$$\log_2 y = \log_2 81^{\log_2 9}$$

$$y = 81^{\log_2 9}$$

$$9^{\log_2 81} = 81^{\log_2 9}$$

$$y = 4^{\log_2 x}$$

$$y = x^{\log_2 4}$$

$$y = x^2$$

6.6 - Solving Exponential and Logarithmic Equations

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Find the inverse

$$y = \left(\frac{2^x + 6}{-3} \right)^{1/3}$$

$$y = \log_2(-3x^3 - 6)$$

Practice

$$1. y = \left(\frac{e^x + 10}{2} \right)^{1/5}$$

$$y = \ln(2x^5 - 10)$$

$$2. y = \log_5(-4x + 6) + 4$$

$$y = \frac{6 - 5^{x-4}}{4}$$

6.6 - Solving Exponential and Logarithmic Equations

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A substance decays 22% each day. After 7 days, there are 9 milligrams of the substance remaining. How many milligrams were there initially?

51.24 *mg*

Anil opened a bank account with an annual interest rate of 4.8% that is compounded annually. He invested \$3,890 in the account at the beginning of 1999 but had to make a withdrawal from his account at the end of 2009 in the amount of \$2,300 to buy a MacBook Pro with no penalty. How much money is in his account at the end of 2025?

\$8,924.68

6.6 - Solving Exponential and Logarithmic Equations

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A new Toyota Camary costs \$36,000 and decreases in value 20% per year. How old is the car (in years) when it is worth half its original cost?

3.1 yrs

Carbon-14 has a half-life of approximately 6,000 years. A sample, originally 100 kg of the material, has decayed through the years and now only 6.25 kg remains. Approximately how old is the sample?

24,000 yrs

